

# Lesson 6

## Doubly Reinforced Beams

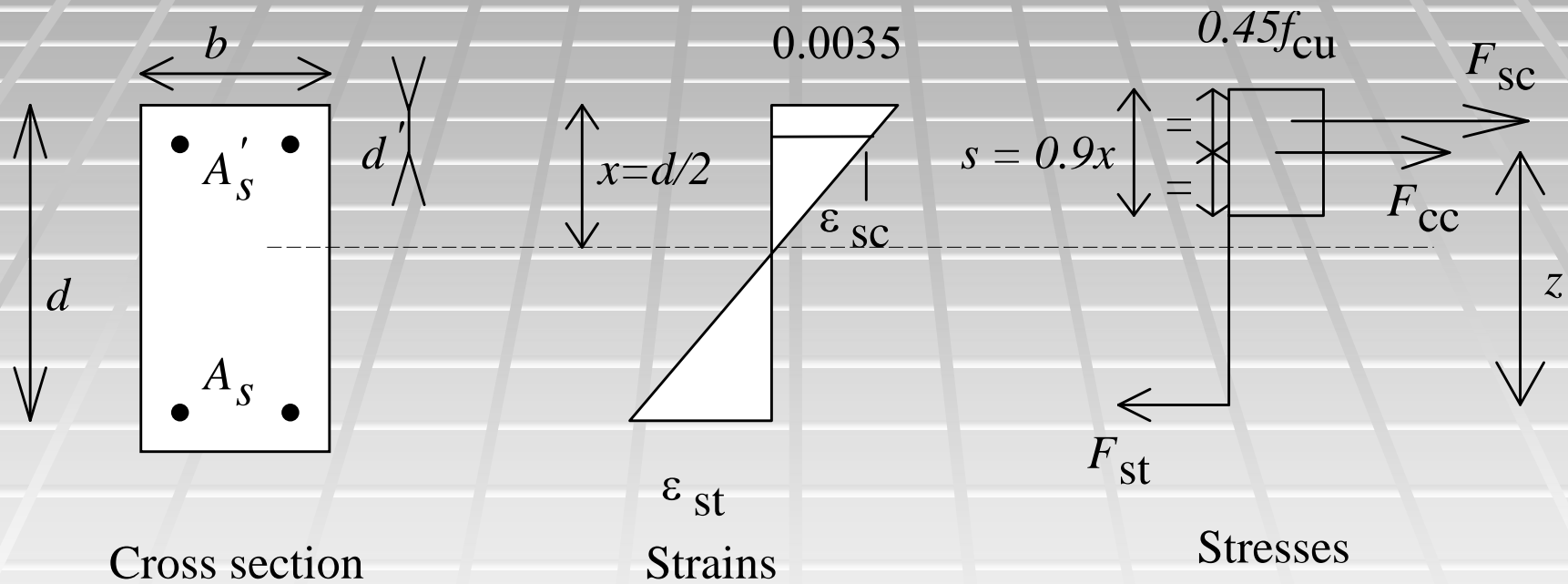
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# Introduction to Doubly R'fed beams

- $M$  – moment calculated from formulae
- $M_u$  – ult. Moment =  $0.156f_c u b d^2$
- If  $M > M_u$ 
  - Concrete can no more withstand the actual moment.
  - Either increase the concrete strength,
  - Or doubly reinforced the beam
- Providing compression steel will resist the moment in excess of  $M_u$

# Stress – Strain blocks



# Area of reinforcement

- Compression steel reinf. Is cal. Using

$$A'_s = \frac{M - M_u}{0.95 f_y (d - d')}$$

d': depth to the compression steel from the top surface

- Tension steel ( $k'=0.156$ )

$$A_s = \frac{M}{0.95 f_y z} + A'_s$$

$$z = d \left[ 0.5 + \sqrt{(0.25 - K' / 0.9)} \right]$$

# Assumption

- Compression steel has yielded i.e. steel stress =  $0.95 f_y$ , but this will only apply if

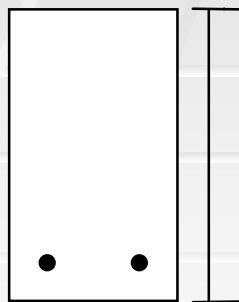
$$\frac{d'}{x} < 0.43 \quad \text{OR} \quad \frac{d'}{d} < 0.215 \quad \text{where } x = \frac{d - z}{0.45}$$

- If  $d'/x > 0.43$  - compression steel will be at a stress less than yield, in which case this stress can be found from the elasto-plastic stress strain curve shown above

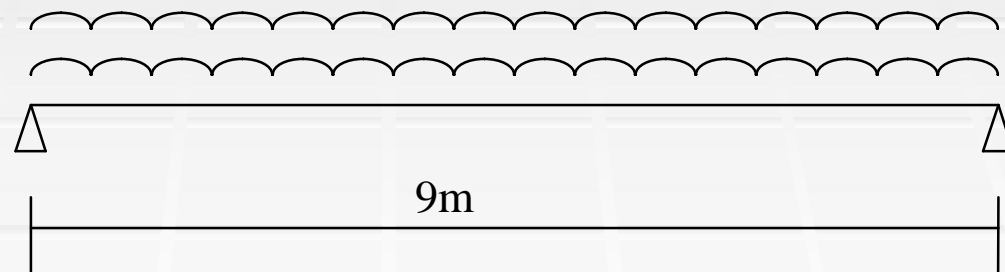
# Worked example

- A simply supported rectangular beam of 9m span carries a characteristic dead ( $g_k$ ) load (inc. Self wt. of beam), and imposed ( $q_k$ ) loads of 6 kN/m and 8 kN/m respectively. Check whether it is a SRB or DRB

$b = 225 \text{ mm}$



$h = 400 \text{ mm}$



$q_k = 8 \text{ kN/m}$   
 $g_k = 6 \text{ kN/m}$

Doubly Reinforced Beam??

# Worked example

- The beam dimensions :  $b= 225\text{mm}$ ,  $h= 400\text{mm}$ ,  $f_{cu} = 30\text{MPa}$  and  $f_y = 460\text{N/mm}^2$  calculate the area of reinforcement required.

$$\begin{aligned}\text{Ultimate load } (w) &= 1.4g_k + 1.6q_k \\ &= 1.4 \times 6 + 1.6 \times 8 = 21.2 \text{ kN / m}\end{aligned}$$

$$\text{Design Moment } (M) = \frac{wl^2}{8} = \frac{21.2 \times 9^2}{8} = 214.65 \text{ kNm}$$

Assuming bar diameter =  $25\text{mm}$

Effective depth  $d = h - 25 / 2 - \text{cover}$

(say  $40\text{mm}$  to main steel inc. link  $\phi$ )  
 $= 348\text{mm}$

$$\begin{aligned}\text{Ultimate Moment } (M_u) &= 0.156f_{cu}bd^2 \\ &= 0.156 \times 30 \times 225 \times 348^2\end{aligned}$$

- Since  $M > M_u$ , doubly r'fed

$$\begin{aligned}&= 127.5 \times 10^6 \text{ Nmm} \\ &= 127.5 \text{ kNm}\end{aligned}$$



# Worked example

## ■ Compression steel calculation

$$d' = \text{cover} + \phi / 2 = 40 + 20 / 2 = 50\text{mm}$$

$$z = d(0.5 + \sqrt{(0.25 - K' / 0.9)})$$
$$= 348(0.5 + \sqrt{(0.25 - 0.156 / 0.9)})$$

$$z = 270\text{mm}$$

$$x = \frac{(d - z)}{0.45} = \frac{348 - 270}{0.45} = 173$$

$$\frac{d'}{x} = \frac{50}{173} = 0.289 < 0.43 \therefore \text{Compression steel has yielded}$$

# Worked example

$$A'_s = \frac{M - M_u}{0.95 f_y (d - d')} = \frac{87.15 \times 10^6}{0.95 \times 460 \times (348 - 50)} = 669 \text{ mm}^2$$

- Choose appropriate reinforcement diameters

$$A_s = \frac{M_u}{0.95 f_y z} + A'_s = \frac{127.5 \times 10^6}{0.95 \times 460 \times 270} + 669$$

# Design methodology

- **a). Calculate K from  $M/fcubd^2$** 
  - $K' = 0.402(bb - 0.4) - 0.18(bb - 0.4)^2$
  - If  $K < K'$ , compression steel is not required, beam shall be designed as a singly reinforced one.
  - If  $K > K'$ , compression steel is required.
- **b). Calculate  $x = (bb - 0.4)d$** 
  - If  $d'/x < 0.37$ , the compression steel has yielded and  $f_{sc} = 0.95f_y$
  - If  $d'/x > 0.37$ , calculate the steel compressive strain  $\epsilon_{sc}$  and hence  $f_{sc}$
- **c). Calculate the area of compression steel,**
  - $A_{s'} = (K - K')fcubd^2/(f_{sc}(d - d'))$

# Design methodology

- **d). Calculate the area of tension steel from**

- $A_s = K'fcubd^2/(0.95fyZ) + A_{sc}'f_{sc}/0.95fy$
- Where  $Z = d - 0.9x/2$

- **e). Links**

- The links should pass round the corner bars and each alternate bar.
- The link size should be at least one-quarter the size of the largest compression bar.
- The spacing of links should not be greater than twelve times the size of the smallest compression bars.
- NO compression bar should be more than 150mm from a restrained bar.