Lecture 3

<u>*Content*</u>

2.1 Introduction to Reinforced concrete beams

Prime purpose of beams - transfer ***loads*** to columns.

Several ***types of RC beams*** - defined with respect to:

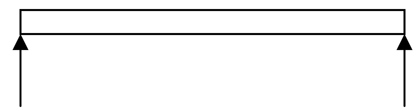
a). Support Conditions,

b). Reinforcement position and

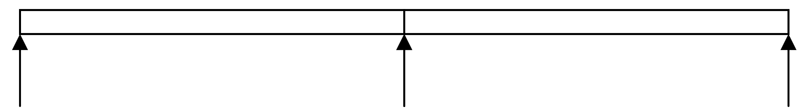
c). Cross-section.

<u>*a). Support Conditions*</u>

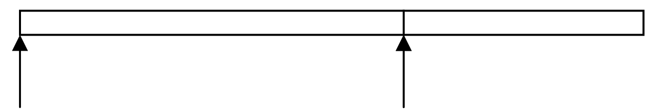
- Simply supported beams,



- Continuous beams and

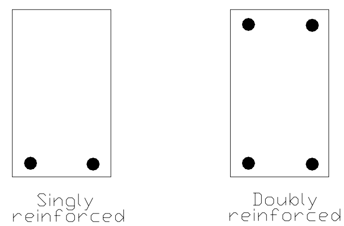


- Cantilever beams.



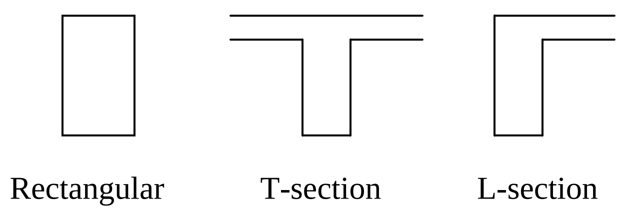
b). Reinforcement position,

- Singly reinforced
- Doubly reinforced



c). Cross-section

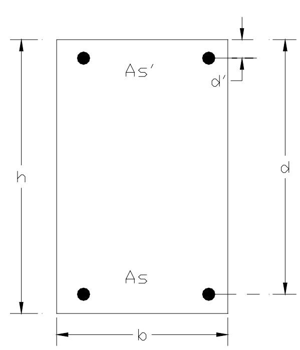
- Rectangular
- T-Section or
- L-Section.



2.2 Singly reinforced beam design

Beams may fail due to (i) excessive bending or (ii) shear – ULS. At SLS, excessive deflection should be avoided.

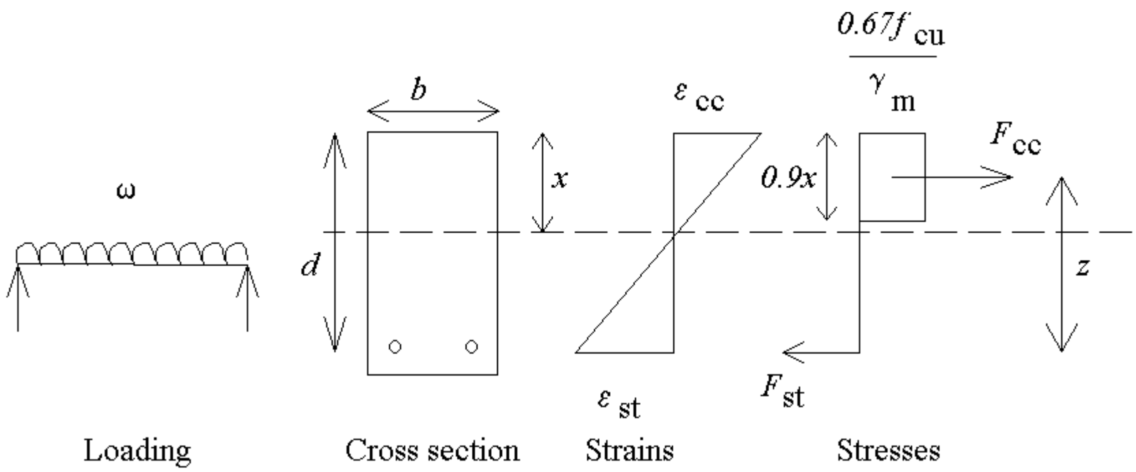
2.3 Notation in beam design



2.4 Analysis of Singly Reinforced Beam

Bending

Consider ***a simply supported singly reinforced rectangular beam***



Load causes - deflection (downwards) - bottom of the beam will be in tension while top in ***compression***.

At ***Neutral Axis – No Tension/Compression***.

Assumption: Plane sections remain plane, the strain distribution will be as shown.

Stress distribution in the concrete varies – depends on magnitude of the applied load.

Ultimate behavior - idealized in # of ways (i) rectangular-parabolic form, (ii) triangular or (iii) much-simplified rectangular block.

The reinforcement provision dictates the mode of failure of beam in bending – Over-reinforced and Under-reinforced.

In an under-reinforced section, since the steel has yielded we can estimate the ***ultimate tensile force*** in the steel.

$$F_{st} = \text{design stress} \times \text{area}$$
$$F_{st} = \frac{f_y A_s}{\gamma_{ms}}$$

Where f_y is the yield stress, A_s the area of reinforcement and γ_{ms} the partial safety factor for the reinforcement **(=1.05**).

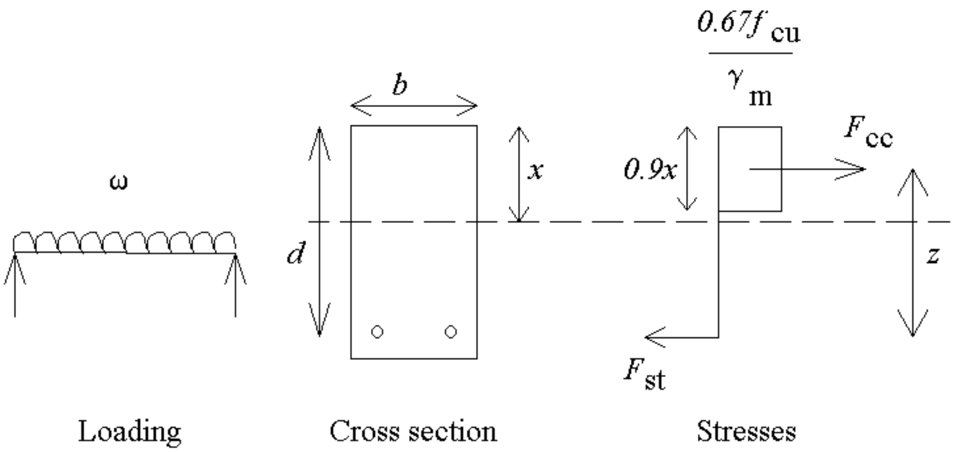
The corresponding stress distribution in the concrete is rather more complicated and has led to the adoption of the simplified rectangular stress block shown. From these assumptions approximate formulae may be derived to quantify

- Ultimate moment of resistance (M_u)
- Area of tension reinforcement (A_s)
- Lever arm (z)

Ultimate moment of resistance (Mu)

The applied loading on the beam gives rise to an Ultimate Design moment (M) on the beam ***in this case at mid-span***. The resulting curvature produces a compression force in the concrete ***F*cc** and a tensile force ***F*st** in the steel. For equilibrium of horizontal forces:

$$F_{cc} = F_{st}$$



The two forces are separated by the lever arm *Z* which enables the section to resist the applied moment and gives the section it's **Ultimate moment of resistance (*M*u).**

For stability:

$$M_u \geq M$$
$$where$$
$$M_u = F_{cc} z = F_{st} z$$

From the simplified stress block above

$$F_{cc} = stress \times area = \frac{0.67 f_{cu}}{\gamma_{mc}} 0.9xb$$
$$and$$
$$z = d - {0.9x}/{2}$$

To ensure that the section designed is under-reinforced it is necessary to place a limit on the maximum depth of the neutral axis (*x*). BS 8110 suggests:

$$x \leq 0.5d$$

substituting for *x, z* and *F*cc in the equation for *M*u above gives:

$$M_u = 0.156 f_{cu} b d^2$$

NOTE: It can be seen from this equation that *M*u depends only upon the properties of the concrete. This means that the concrete alone determines the maximum moment carrying capacity of a section ($M \leq M_u$).

If a greater moment capacity is called for it can only by obtained by either increasing the size of the section/concrete strength or alternatively by DOUBLY REINFORCING

Area of Tension Reinforcement (As)

At the limit when $M_u = M$

$$M = F_{st} z = \frac{f_y A_s}{\gamma_{ms}} z$$

rearranging and putting $\gamma_{ms} = 1.15$

$$A_s = \frac{M}{0.87 f_y z}$$

The solution of which requires an expression for *z*.

Lever arm, *z*.

At the limit when $M_u = M$

$$M = F_{cc}z = \frac{0.67 f_{cu}}{\gamma_{mc}} 0.9bxz$$

$$\text{putting } \gamma_{mc} = 1.5$$

$$= 0.4 f_{cu} bxz$$

$$= 0.4 f_{cu} bz 2\frac{(d-z)}{0.9}$$

$$= \frac{8}{9} f_{cu} bz(d-z)$$

$$\text{dividing both sides by } f_{cu}bd^2$$

$$\frac{M}{f_{cu}bd^2} = \frac{8}{9}(z/d)(1 - z/d)$$

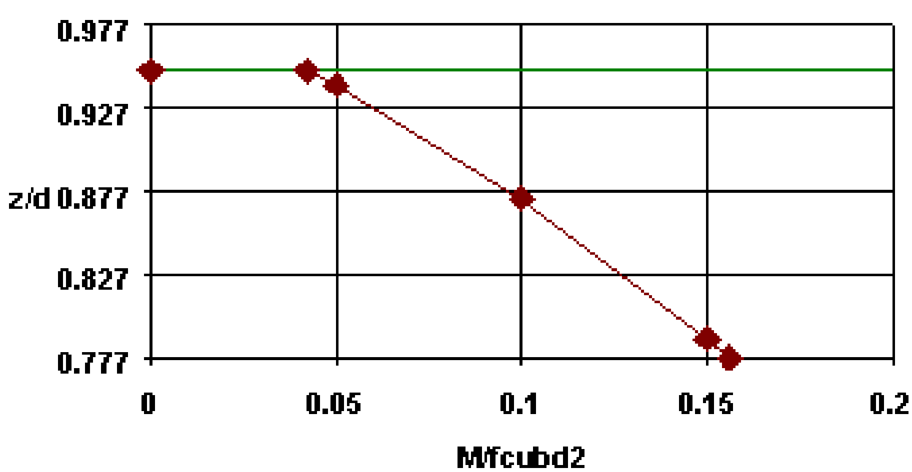
$$\text{Subs } K = \frac{M}{f_{cu}bd^2} \text{ and } z_0 = z/d \text{ gives:}$$

$$K = \frac{8}{9}(z_0)(1 - z_0) \text{ or } 0 = z_0{}^2 - z_0 + 9K/8$$

This is a quadratic and can be solved to give

$$z_0 = z/d = 0.5 + \sqrt{(0.25 - K/0.9)}$$

which can be used to draw a lever arm curve



OR re-arranged to give a function for z in terms of d

$$z = d(0.5 + \sqrt{(0.25 - K/0.9)})$$

Having found *z* we can substitute in the equation for *A*s .

It must however be remembered that these simplifying equations may only be used to determine *A*s when $M \le M_u$.

In summary design for bending requires the calculation of the maximum design moment *M* and the ultimate moment of resistance of the section *M*u. If $M \le M_u$ then only tension reinforcement is required and the area of steel can be calculated from:

$$A_s = \frac{M}{0.87 f_y z}$$

making use of the graph or expression $z = d(0.5 + \sqrt{(0.25 - K/0.9)})$ to find z.

Where $M > M_u$ ***the designer must consider either a DOUBLY REINFORCED section or increased section capacity (d and fcu).***

2.5 Design methodology

Step 1: Dead load calculations

Step 2: Live Loads

Determine the live loads from the respective BS 6399: Part 1: Loadings for buildings.

Step 3: Determine the load at ULS from the following load combination.

'@ULS, load = 1.4Dead + 1.6Live

Step 4: Sketch of beam being designed

Step 5: Determine the Maximum moment at Mid-span

Step 6: Calculate the moment coefficient k from $M/f_{cu}bd^2$

If k < 0.156, no compression steel is required. Beam should be designed as a singly reinforced beam.

Step 7: Calculate the lever arm Z form d[0.5+ sqrt(0.25-(k/0.9))] < or = 0.95d

Step 8: Calculate the area of tension steel, A_s from $M/0.95f_yZ$

Step 9: Calculate the minimum area of steel required by the section from

$$\mathbf{A_{s\ min} = 0.13\%bh}$$

Step 10: $A_{s\ required}$ = Maximum ($A_{s\ calculated}$, $A_{s\ min}$)

Step 11: Select suitable reinforcement bars.

NB: The Area of steel provided should always ***be equal or greater than 0.13 for high tensile steel*** and ***0.24 for mild steel*** but ***less or equal to 4.0***

2.6 Group Assignment (To be submitted on or before Lecture 4)

A simply supported rectangular beam of 7m span carries a characteristic dead, g_k (inclusive of the beam's self-weight) and imposed, q_k, loads of 12 and 8 kN/m respectively.

Given the following data:

Beam: 275mm wide and 500mm high, Reinforcement steel: High tensile steel, Concrete grade: C30, Exposure condition: Mild and Fire resistance = 1.5h.

Design the rectangular beam according to BS8110.

[Answer: 1411mm^2][11 marks]